Recursion - II

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Q:→ Find the recurrence relation satisfying

$$y(k) = (A + Bk)4^k$$

sol:→

$$y(k) = A(4)^k + k \cdot B(4)^k$$

$$y(k+1) = A(4)^{k+1} + (k+1)\cdot B(4)^{k+1}$$

$$= 4\cdot A(4)^k + 4(k+1)\cdot B(4)^k$$

$$y(k+2) = A(4)^{k+2} + (k+2) B(4)^{k+2}$$

$$= 16\cdot A(4)^k + 16(k+2)\cdot B(4)^k$$

i.e

$$y(k) = A(4)^k + k\cdot B(4)^k$$

$$y(k+1) = 4\cdot A(4)^k + 4(k+1)\cdot B(4)^k$$

$$y(k+2) = 16\, A(4)^k + 16(k+2)\cdot B(4)^k$$

$A(4)^k \to x$

$B(4)^k \to y$

$$\begin{vmatrix} y(k) & 1 & k \\ y(k+1) & 4 & 4(k+1) \\ y(k+2) & 16 & 16(k+2) \end{vmatrix} = 0$$

$$y(k)\,(64(k+2) - 64(k+1)) - y(k+1)\,(16(k+2) - 16k) + y(k+2)\,(4(k+1) - 4k) = 0$$

$$\Rightarrow y(k)\cdot 64 - y(k+1)\cdot 32 + y(k+2)\cdot 4 = 0$$

$$\Rightarrow 16\,y(k) - 8y(k+1) + y(k+2) = 0$$

$$\Rightarrow y(k+2) - 8y(k+1) + 16y(k) = 0$$

Homo Linear Recc. relation order 2.

* Solution of linear Recurrence Relation with constant

Solution of Linear Recurrence Relation with constant coefficients.

→ Linear Recc. Relation with constant Coefficients

Consider a reccurence relation as

$$c_0 f(n) + c_1 f(n-1) + c_2 f(n-2) + \ldots + c_k f(n-k) = q(n) \quad \text{—— (1)}$$

where c_i's are constant.

is known as linear recurrence relation with constant coefficients.

a) If $q(n) = 0$ then (1) is called

Homo Linear Recc. Relation with constant coeffs.

b) If $q(n) \neq 0$ then (1) is called

Non Homo Linear Recc Relation with constant coeffs.

Characteristic Equation :→

Consider a linear recc. relation with constant coefficients as

$$c_0 f(n) + c_1 f(n-1) + c_2 f(n-2) + \ldots + c_k f(n-k) = q(n) \quad \text{—— (2)}$$

where c_i's are constant

Then the equation

$$c_0 a^k + c_1 a^{k-1} + c_2 a^{k-2} + \ldots + c_k = 0$$

is known as Char. eqn of (2)

n

which is obtained by replacing $f(n) = a^n$ in homo. part of (2).

$$c_0 a^n + c_1 a^{n-1} + c_2 a^{n-2} + \dots + c_k a^{n-k} = 0$$

$$\Rightarrow c_0 a^k + c_1 a^{k-1} + c_2 a^{k-2} + \dots + c_k = 0$$

(by dividing with a^{n-k})

e.g (1) Consider the recurrence relation

$$f(k) - 6f(k-1) + 8f(k-2) = 3$$

Take $f(k) = a^k$ in the homo part of the above recc. relation

$$a^k - 6a^{k-1} + 8a^{k-2} = 0$$

Divide by a^{k-2}

$$a^2 - 6a + 8 = 0$$

(2) Consider the recurrence relation

$$y(n+2) + y(n+1) - 12y(n) = 0$$

Take $y(n) = a^n$ in the above recc. relation

$$a^{n+2} - a^{n+1} - 12a^n = 0$$

Divide by a^n

$$a^2 - a - 12 = 0$$

which is the required char eqn.

Solution of Homo Linear Recc. Relation with Constant Coefficients using Char. Eqn

Consider a homo linear recc. relation with constant coefficient of order k

$$f(n) + c_1 f(n-1) + c_2 f(n-2) + \dots + c_k f(n-k) = g(n) \quad \text{—} (1)$$

I) Write down the char eqn of (1)

$$a^k + c_1 a^{k-1} + c_2 a^{k-2} + \dots + c_k = 0$$

(order of Rec. relation = degree of char eqn)

II) Find the roots of char eqn say $a = m_1, m_2, \dots, m_k$

a) If the characteristic eqn has k distinct roots then

$$f(n) = A_1 m_1^n + A_2 m_2^n + \dots + A_k m_k^n$$

b) If the characteristic eqn has j repeated roots i.e $m_1 = m_2 = \dots = m_j$ (= m say) then

$$f(n) = (A_1 + nA_2 + n^2 A_3 + \dots + n^{j-1} A_j) m^n + A_{j+1} m_{j+1}^n + \dots + A_k m_k^n$$

c) If the characteristic eqn has imaginary roots say $\alpha \pm i\beta$. Then

$$f(n) = A_1 (\alpha + i\beta)^n + A_2 (\alpha - i\beta)^n$$

Ex:→ Find char roots of the recurrence relation.

(i) $Q(k) + 2Q(k-1) - 3Q(k-2) - 6Q(k-4) = 0$

ii) $T(k) - 7T(k-2) + 6T(k-3) = 0$

iii) $S(k) - 2S(k-1) + 4S(k-2) = 0$